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Dynamic Productivity, Efficiency, and Technical Innovation in Education: A Mathematical Programming Approach Using Data Envelopment Analysis

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ABSTRACT

Despite the long-standing debate surrounding the performance of the U.S. public education system, there is an absence of comprehensive empirical evidence on this issue across different states. Initial dialogues on educational performance prescribed overall increases in expenditures for securing productivity gains. However, more recent themes emphasize the importance of productive efficiency as a key factor in securing performance increases.

This paper measures technical efficiency and total factor productivity in educational production units that utilize a multioutput production technology. The technical efficiency scores are explicitly conditioned on socioeconomic and environmental influences. Particular attention is paid to the specification of varying returns to scale and input disposabilities in a piecewise linear technology. An empirical application to Utah school districts reveals that filtering out scale, congestion, and socioeconomic components from technical efficiency measures, provides superior pure technical efficiency estimates. Similarly, disentangling scale, congestion, and technological innovation components in the total factor productivity measure of Utah schools, observed over the period 1993-95, provides more precise estimates of dynamic productivity.

We find evidence of high levels of pure technical efficiencies across Utah school districts but weak evidence of technological innovations. The implication is that Utah schools are overall technically efficient, and policies seeking broader and better educational outcomes need to be focused on correcting for inefficient scales and research expenditures that would shift out the technical frontier. Thus, the widespread "efficiency or expenditure" debates, as alternative sources of educational productivity increases, may not be well posed. This study demonstrates empirically that efficiency and expenditure alignments are not mutually exclusive.

JEL Classification: D20, C14, R30
Key words: technical efficiency, dynamic productivity index, education, frontier shifts, nonparametric, DEA

DYNAMIC PRODUCTIVITY, EFFICIENCY, AND TECHNICAL INNOVATION IN EDUCATION: A MATHEMATICAL PROGRAMMING APPROACH USING DATA ENVELOPMENT ANALYSIS[1]

The efficacy of public education in the United States has been a source of considerable concern and debate over several decades. The accountability of public education is especially important now in a milieu of spiraling costs, increasing population, and the apparent inability of administrators to improve educational outcomes by augmenting expenditures. In the wake of widespread disenchantment regarding public education and public services in general, policy makers have vigorously renewed their commitment towards the performance assessment of public school units.

Historically, the Coleman Report (Coleman et al. 1966) advanced equalization policy reforms to smooth heterogeneity in expenditure budgets and endorsed equal educational opportunities across school districts. The ensuing reforms in property tax revenues, which largely forge public school expenditures, secured homogenous funding and expenditure structures. However, with increasing evidence that expenditures were not the driving variable behind educational performance, it was not entirely clear that adjustments in expenditures necessarily map into performance increases. Consequently, the homogeneous structure only shifted the burden of adjustment from expenditure alignments to the assessment of relative performance among the homogenous school units. The publication of *A Nation at Risk* (National Commission on Excellence

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in Education 1983) heralded these concerns and reignited the debate surrounding public education in the United States. The renewed dialogues emphasized the importance of productivity and efficiency as key factors in the public education reform movement.

While particularly intense discussions have been focused on the ability of schools to adequately equip students for competing in higher institutions and labor markets, the primary question facing policy makers is whether the U.S. public education system efficiently develops students' cognitive skills as measured by standardized test scores. In response to this question, there has spawned a fairly large amount of literature on the estimation of relative performance across school districts in different states. Typically, a school unit is regarded as an optimizing firm producing an output of student learning by using educational and socioeconomic inputs. As a nonprofit organization, its behavioral principle is assumed to be the efficient use of feasible inputs to produce given levels of output without resource wastage (or maximum outputs from given resources). Consequently, performance across schools may be ranked on the basis of efficient production schedules. However, as performance or output is constrained by limited resources, there exists an upper bound beyond which performance cannot increase. Conventional wisdom proposes this upper bound as a primal transformation function describing the production technology. The primary question to policy makers is thus, to decipher where an observed school unit is located relative to this primal frontier and what policy implications are dictated by such location.

Although the efficiency and productivity of public education raises important concerns and needs for policy action, there have been no comprehensive empirical studies across states that effectively model these diverse concerns. In light of large increases in incomes and employment and under pressure in recent years from a growing population, the state of Utah faces a significant increase in the demand for a better and larger public education system. Given that property taxes remain the dominant source of funding for public education in Utah, property owners face serious risks from the introduction of new and increased taxes in order to fund the increasing demand for schools. More importantly, as taxes are often undesirable and are not pareto-efficient, policy makers must carefully consider the efficiency of existing schools to investigate possible increases in performance before sanctioning additional funding. As such, if schools are inefficient, then additional funds may actually accentuate impoverished performance rather than contribute to output. Thus, in order to develop a cogent plan for improving the state of public education, inefficiencies and sources of the resulting impoverished performances need to be identified and the effectiveness of overall increases in expenditures per pupil evaluated in the light of this information.

Only one study (Chakraborty, Biswas, and Lewis 1996), assessing performances across Utah school districts, could be identified. However, this study has not adequately modeled the diverse concerns related to the public education system in Utah and many important issues remain unexplored. First, the production function for education may be mispecified in this conventional single-output model as the realization of skills from education is not singular. Second, the efficiency model is overly restrictive in parametrization of the production technology and in assumptions concerning returns to scale. Third, the existing study fails to model fixed and weakly disposable inputs correctly which leads to biased efficiency estimates. Such considerations are especially important in the context of education as efficiencies of schools are highly conditioned on the socioeconomic environment of students and the qualifications of teachers. Finally, there exists clear theoretical evidence in a broader development literature that efficiency issues are inherently dynamic and that dynamic structure offers important policy insights for extension or research activities. Despite this, the existing study has maintained a static conceptualization of educational efficiency.

Thus, this study is a first attempt at assessing the performance of Utah school districts using a comprehensive approach that addresses the concerns listed above. Dynamic modeling techniques are applied to a multioutput educational production process. The use of an input-based approach seems more appropriate for public schools that generally do not operate by profit-maximization or cost-minimization choice rules. We estimate magnitudes of technical (in)efficiency and its components (i.e., scale inefficiency and input congestion) for a panel of 36 Utah school districts. In deriving both radial and nonradial technical efficiency estimates, we pay particular attention to the fact that efficiencies across schools are conditioned on socioeconomic and environmental factors that are fixed and exogenous to the school unit. Thus, efficiencies are estimated within the framework of both fixed and variable inputs, socioeconomic factors, congesting inputs, multiple outputs, and varying returns to scale structures. We also estimate intertemporal frontiers, which yield estimators of total factor productivity across the time periods. This study separately identifies sources of productivity change, such as shifts in the frontier (technical innovation), changes in technical efficiency, changes in scale efficiency, and changes in congestion during the period 1993-95. Careful analysis of the efficiency and dynamic productivity estimates with the interpretive decompositions derived in this study should yield a major advance in policy makers' understanding of the educational production system.

II. Key Issues

School units convert various instructional and noninstructional inputs into multiple-learning outputs as measured by student achievement test scores (e.g., standardized math, verbal, or science test scores). Inputs, such as the availability of teachers per student, the proportion of teachers with advanced degrees, and expenditures per student, affect output directly and often are endogenous to the school unit. These inputs are called discretionary or controllable inputs. Nondiscretionary inputs, which include various socioeconomic or environmental factors such as family income and the assessed property value per student, affect learning indirectly and usually are beyond the school unit's control. The multiple-learning outputs produced by schools have been observed, both analytically and empirically, to be increasing functions of both the instructional and noninstructional inputs. The key issues surrounding efficiency or relative performance assessment of school units are primarily driven by stylized facts of the educational production function. These considerations, which motivate the empirical methodology adopted in this paper, are discussed below.

First, theoretical production functions are extremal and define the maximum output possible from given inputs or, alternately, the minimum amount of inputs necessary to produce a given level of outputs. The concept of maximality or minimality inherent in these functions sets a limit to the range of feasible observations. However, conventional OLS-type conditional expectation estimators approximate the average production correspondence instead of estimating the production frontier surface. In order to estimate the extremal production function we use the mathematical programming approach based on the activity analysis model of production. This nonparametric approach is due to the pioneering works of Farrell (1957) and Koopmans (1951) and was formalized as the data envelopment analysis (DEA) approach from works by Charnes, Cooper, and Rhodes (1978) and Fare, Grosskopf, and Lovell (1985).

A second consideration in modeling efficiency in school districts is the appropriate treatment of socioeconomic factors, which have important effects on productivity. While socioeconomic and environmental variables are exogenous to the school unit, excluding them from the model leads to specification errors and a biased technical efficiency component in the productivity measure. However, these variables, when included, need to be treated differently from endogenous variables in the system. Ideally, efficient school units should be able to bring forth high educational outcomes for students even in relatively low socioeconomic conditions and poor environments. These concerns are similar to those faced when modeling fixed factors. Fixed factors, when treated as variables, generate upward biases in technical efficiency estimates and need to be modeled carefully. We model socioeconomic factors nonparametrically by using a modified form of the DEA model, where efficiency of schools is defined over a subvector of the (endogenous) variables for given levels of the vector of socioeconomic and environmental variables. Additionally, parametric estimates, using a second-stage regression of the DEA efficiency scores on the socioeconomic and environmental variables, are derived to provide insights into the nature of socioeconomic and environmental influences.

A third key issue in modeling educational frontiers relates to the typical restrictions imposed on production technologies. Standard assumptions specify inputs as freely disposable, implying that increases in any of the inputs do not lead to decreases in output. Thus, input-based efficiency targets are always approached by input reduction. However, in the context of this study, we suspect that overutilization of teaching personnel with advanced degrees may hamper student learning. These inputs then have to be modeled and evaluated in their contribution towards efficiency under weak disposability assumptions, such as increases in this input, and holding all other input's usage constant may generate decreases in output. Equally burdensome is the use of restrictive returns-to-scale assumptions which prevent the separate identification of scale and technical inefficiencies. Given the importance of uncovering different types and sources of inefficiency for policy making, efficiency estimates need to be decomposed into scale, input congestion, and pure technical inefficiency components.

Moreover, consider the technically efficient expansion path of a school unit as a series of movements from within an output set, which finally push it onto the frontier where it is considered efficient. In a multiperiod time horizon, these snapshot descriptions may be combined to redefine the efficient school as one that undergoes the continuous motion of moving from its initial location within the output set onto a frontier which itself shifts over time due to the salutary effects of technical innovations. In a dynamic world with changing technical and economic environments, efficient school units constantly need to adjust for new equilibria, and high payoffs exist for managerial effectiveness and increases in information. Thus, efficiency measures should provide policy information on both the statics and dynamics of school performance.

III. Methods[2]

The Basic DEA Model

The input-based frontier estimators in DEA construct a nonparametric, piecewise, linear surface by enveloping the sample data with a convex hull consisting of a series of linear segments. The constructed reference surface provides an upper bound for technical efficiency as school units hitting this bound would be fully efficient. Although, the constructed technology is well behaved and satisfies general axioms of production theory (Fare, Grosskopf, and Lovell 1994), it is not differentiable everywhere due to the presence of linear segments which connect the best practice units. However, asymptotic smoothness is achieved as the number of activities increase and the piecewise representation converges to the smooth neoclassical function.

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[2]The empirical strategies followed in this study are drawn from the theoretical discussion provided in fare, Grosskopf, and Lovell 1994.

Consider the activities of I school units, each employing N inputs to produce M outputs. Let N denote the $(I \times N)$ matrix of N inputs used by I school units, with a typical element x_{in} denoting the nth input utilized by the ith school unit. Let $\boldsymbol{M}$ represent the $(I \times M)$ matrix of M outputs of I different school units, where the typical element y_{im} denotes the mth output of the ith school unit. Outputs and inputs are hypothesized to obey the usual nonnegativity restrictions. The piecewise linear input set, constructed under standard assumptions of constant returns to scale and free disposability of inputs, denotes all input vectors capable of producing at least output vector y:

$$L(y \mid C, F) = \{x : y \leq zM,\ zN \leq x,\ z \in \Re^I_+\},\ y \in \Re^M_+ \tag{1}$$

where z denotes an $(1 \times I)$ intensity vector that forms convex combinations (with VRS) of observed input and output vectors. The technical efficiency of an observed school unit is measured by a Shephard distance function (1953; 1970) measured from the candidate input vector towards the constructed piecewise linear, convex isoquant. The distance function measure seeks out a parameter of technical efficiency ξ such that, when multiplied to an inefficient school's input bundle, renders that school efficient. The input-based distance function measure, bounded by 0 and 1, can be written as:

$$D_i(y^i,\ x^i \mid C, F) = MIN\{\xi : \xi x^i \in L(y^i \mid C, F)\} \tag{2}$$

This input-oriented measure considers the equiproportionate shrinkage in inputs required to project a school back onto the frontier, while still maintaining the production of its given level of outputs. Solution of the following linear programming model consolidates equations (1) and (2) and obtains school-specific technical efficiency measures relative to the bounding technology:

$$B_i(y^i, x^i \mid C,\ F) = \underset{\xi,\ z}{MIN}\ \xi \tag{3}$$

subject to

$$\begin{aligned} y^i &\leq zM \\ zN &\leq \xi x^i \\ z &\in \Re^1_+ . \end{aligned}$$

Figure 1 describes the input requirement set for a sample of 6 (I = 6) schools situated at points A, B, C, D, E, and F. Application of the DEA approach finds schools A, B, C, and D to be efficient. These efficient schools are used to construct an envelopment surface over the sample schools in a manner that ensures that all schools are either on or below the envelopment frontier. A close observation of Figure 1 immediately identifies schools E and F to be inefficient.

In order to have individual efficient target locations for each unit, DEA constructs i virtual "super" schools which are efficient. As an example, we illustrate the construction of a super school F* for inefficient unit F. Illustrating for the first input, efficiency in input use of the super school is guaranteed by the inequality in (1)

$$z_A x_{A1} + z_B x_{B1} + \ldots z_J x_{J1} \leq x_{J1} \tag{4}$$

The left-hand side of the previous inequality, F*, is a composite input bundle formed as a linear combination of input bundles of all other relevant school units. F* represents the efficient reference point for F and serves as a direct efficiency comparator. In (4), F* is constructed as a linear combination of other relevant schools with weights (z_i) assigned on the basis of this relevancy. Thus, F* serves as an efficiency target location onto which school unit F can be projected by an equiproportionate shrinkage in its input vector. This is given by (3) where ξ, evaluated at the optimum, is a number less than unity by which inputs of F can be multiplied while still producing at least as much of the given level of outputs. Thus, ξ provides a relative measure of technical efficiency as the maximum feasible shrinkage of the input vector possible while maintaining production at given output level y.

Although the model discussed above served as the basis of this efficiency analysis, in order to derive robust efficiency estimates, the basic DEA model is modified to take into account the idiosyncrasies of educational inputs. Towards this end, we construct several submodels to address the key issues (section 2) surrounding the educational production function.

Conditional Efficiency Estimates: The Subvector Efficiency Model

Given the nature of exogenous socioeconomic inputs, we use estimation methods, which allow for unbiased efficiency estimates while considering the impact of socioeconomic factors that are fixed and beyond the school units control. Additionally, the more conventional two-step method of regressing the efficiency scores from the basic DEA model on the exogenous socioeconomic inputs is used to give an account of the exogenous influences.

The subvector DEA efficiency measure allows a subvector of (endogenous) inputs to be scaled back onto the frontier while recognizing the fixed nature of the socioeconomic variables. Letting x_v represent variable inputs under the school's control and x_K represent the fixed socioeconomic inputs, we partition the input matrix N into variable and fixed input vectors ($N = N^\mathrm{v}$, N^K). The subvector technology under variable returns and free input disposability may be formed as:

$$L(y\,|\,V, F) \;=\; \{x_\mathrm{v}, x_\mathrm{K} : y \le zM, zN^\mathrm{K} \le x_\mathrm{K}, zN^\mathrm{v} \le x_\mathrm{v}, \Sigma z_i \;=\; 1, z \in \Re^I_+\},\; y \in \Re^M_+ \tag{5}$$

The radial subvector technical efficiency measure is given by:

$$S_i(y^i, x^i_\mathrm{v}, x^i_\mathrm{K}|V, F) \;=\; MIN\{\xi : (\xi x^i_\mathrm{v}, x^i_\mathrm{K}) \in L(y^i|\,V, F)\} \tag{6}$$

School-specific radial subvector technical efficiency estimates can be derived by solving the following linear program I times:

$$S_i(y^i, x^i_\mathrm{v}, x^i_F\,|V, F) \;=\; MIN_{\xi, z}\xi \tag{7}$$

subject to

$$\begin{gathered} y^i \le zM \\ zN^\mathrm{v} \le \xi x^i_\mathrm{v} \\ zN^k \le x^i_k, \\ z \in \Re^I_+ \\ \Sigma z_i = 1. \end{gathered}$$

$S_i(y^i, x_v^i, x_\kappa^i|V, F)$ is the technical efficiency measure of the inputs under the school unit's control for given amounts of the socioeconomic inputs.

The parametric approach towards accounting for socioeconomic and environmental influences utilized a Tobit regression (as the input-based efficiency scores were bounded between 0 and 1) of DEA efficiency scores on the socioeconomic factors for consistent and unbiased estimation. The residuals obtained from this regression are the "pure" technical efficiency scores after eliminating the effects of the uncontrollable inputs. While the zero values for the residuals imply that a school district's performance is as good as the average school district with the same socioeconomic variables, the nonzero values signify differences in the performance from the average school district with the same set of uncontrollable factors. Higher values of the Tobit residuals signal higher technical efficiencies given the socioeconomic and environmental factors.

Priors on the scales of production are often arbitrary and impose a large amount of structure, hence, leading to biased efficiency measures. This destroys one of the basic virtues of the nonparametric approach and leads us back to the parametric problem of "incredible" restrictions. Moreover, given the fact that conditional efficiency estimates identify technical efficiency and socioeconomic components separately, a further confounding component in the technical efficiency measure could be scale inefficiencies. In order to identify scale inefficiencies, we relax the constant returns-to-scale assumption on the frontier and allow for a highly flexible technology which accommodates variable, decreasing, and nonincreasing returns to scale. For example, constraining the sum of the intensity vector, *z*, to be equal to unity allows for variable returns to scale. The earlier nonnegativity constraint on *z* in (3) allowed all observations to be scaled up or down, effectively imposing constant returns to scale.

Input Overutilization: The Congestion Model

The standard frontier technology is constructed as the constant returns to scale, convex, free disposable hull of observed input and output vectors. Free disposability implies that increases in any of the inputs do not lead to a decrease in output. In terms of (1), this can be written as:

$$x \geq x' \in L(y \mid C, F) \Rightarrow x \in L(y \mid C, F) . \tag{8}$$

However, given the earlier hypothesis about the overutilization of qualified teachers, we need to define the technology under the weaker restriction of weak disposability, i.e., if increases in inputs are not proportional, then output may decrease:

$$x \in L(y \mid C, W) \Rightarrow \alpha x \in L(y \mid C, F) , \; \alpha \geq 1 . \tag{9}$$

The failure to correctly identify the nature of disposability of an input would incorrectly attribute the school's deviation from the frontier, caused by overutilization of highly qualified teachers, to technical inefficiency. This is demonstrated in Figure 1, where the efficiency of school *W* is evaluated relative to the lower bound *ABCD* of the input set constructed under free disposability assumptions. Now suppose X2 represents the teacher's degree input, then the constructed technology implies that increases in teachers with advanced degrees, holding other inputs constant, allows the school to remain on isoquant *ABCD* producing the same level of output. Thus, school *W*, which lies outside the free disposable isoquant, is found to be technically inefficient as measured by a ray from the origin. However, this is incorrect if teachers with advanced degrees cause intellectual overcrowding and would lead to identification of congestion as technical inefficiency.

We use an input-based technical efficiency measure relative to a technology that exhibits variable returns to scale and two types of input disposability behavior. The technical efficiency

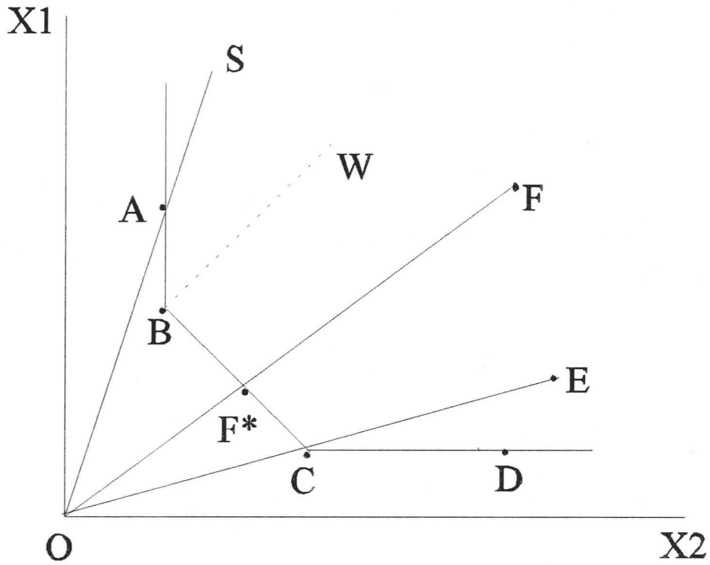

Figure 1. A Piecewise Linear Convex Frontier.

relative to this technology is defined over the subvector of inputs under the school's control as before. Let x_v represent the freely disposable variable inputs under the school's control, x_κ represent the free disposable socioeconomic variables, and $x_{\tilde{\omega}}$ represent the weakly disposable input vector (teacher's degree). Thus, the input matrix N is partitioned into 3 subvectors ($N = N^v, N^\kappa, N^{\tilde{\omega}}$). Using the variable returns to scale, piecewise linear input set for the endogenous inputs, x_v, and exogenous inputs, x_κ, and with free disposability of all input vectors excepting for input subvector $x_{\tilde{\omega}}$, the school-specific radial subvector technical efficiency estimates can be derived by solving the following linear program I times:

$$Sw_i(y^i,\ x^i_\omega,\ \kappa^i_v,\ x^i_k | \, V, F^{\kappa,v}, W^\omega) = MIN_{\xi,z}\ \xi \tag{10}$$

subject to

$$\begin{aligned} y^i &\le zM \\ zN^\kappa &\le x^i_\kappa \\ zN^v &\le \xi x^i_v \\ zN^\omega &= \xi x^i_\omega \\ z &\in \Re^I_+ \\ \Sigma z_i &= 1. \end{aligned}$$

Input Slacks: The Nonradial Conditional Efficiency Model

In the models presented above, efficiency is measured radially. Technology is modeled with the input correspondence $y \rightarrow L(y)$. Given that inputs x are feasible, i.e., $x \in L(y)$, input-based technical efficiency is measured by determining the location of x in the input requirement set, $L(y)$. Radial efficiency measures seek out the maximum feasible shrinkage necessary to project an observed input vector onto the isoquant. However, this equiproportionate shrinkage along the radial path from feasible x^i back towards the origin does not take into account input slacks and is unable to reflect optimal input usage. For example, in Figure 1, consider the input bundle S (i.e., school S). A radial shrinkage measure projects S onto the efficiency target, A, on the isoquant. However, the true efficiency target for S is B, as it exists on the same isoquant but uses AB amount (slack) less of input $X1$. To be able to project back input vectors onto such efficient subsets of the isoquant, one needs disproportionate reductions in inputs. This is achieved by the nonradial Russell efficiency measure, which assigns efficiency labels only to vectors which belong to the efficient subset. The nonradial Russell measure for constant returns to scale and free disposability assumptions is:

$$NR_i(y^i, x_v^i, x_\kappa^i | C, K) = MIN_{\xi, z} \Sigma_{n=1} N \xi_n / N \tag{11}$$

subject to

$$\begin{aligned} y^i &\leq zM \\ zN^\kappa &\leq x_\kappa^i \\ zN^v &\leq \xi x_v^i \\ z &\in \Re_+^I \end{aligned}$$

where ξ is a (N X 1) vector.

In Figure 1, radial and nonradial efficiency measures for school F coincide, as there is no slack in the efficiency target point F^*. For school S, with slack in input $X1$, the nonradial measure gets past the slack by comparing x^i to $(\xi_1^* x_{i1}, \xi_2^* x_{i2}, \ldots \xi_N^* x_{iN})$.

Productivity Change, Efficiency Change, and Technological Innovations: The Dynamic Model

The above measures give only a static description of efficiency. Dynamic productivity and efficiency estimates can be derived using multiperiod analysis. However, in this context, standard productivity indexes, such as the Fisher and Tornqvist indexes, require information on prices and impose profit maximization behavioral assumptions, which are incredible for the educational firm. We use Malmquist productivity indexes, which use only information on the input and output quantities, to estimate productivity and productivity growth over time. The Malmquist productivity index, proposed by Caves, Christensen, and Diewert (1982), measures productivity differences across time periods and, in its modified version (Fare, Grosskopf, and Weber 1989), describes sources of dynamic productivity changes. These indexes are based on distance functions, which rely on the primal description of technology. At each time period t = 1, 2 . . . T, technology is modeled as:

$$L^t(y_t | C, F) = \{x_t : y_t \le zM_t, zN_t \le x^t, z \in \Re^J_+\}, y^t \in \Re^M_+. \quad (12)$$

The Malmquist input-based productivity index measures changes in performance during two time periods and can be expressed as:

$$M_i^{t+1}(y^{t+1}, x^{t+1}, y^t, x^t|C, F) = \left[\frac{S_i^t(y^t, x^t|C, F)}{S_i^t(y^{t+1}, x^t|C, F)} * \frac{S_i^{t+1}(y^t, x^t|C, F)}{S_i^{t+1}(y^{t+1}, x^t|C, F)}\right]^{1/2}.$$

The Malmquist index is thus computed by running each of the four linear programs I times. The same period efficiency measures, such as the numerator of the first expression and the denominator of the second expression, are the same as (1) with observations dated in order to specify year-specific input correspondences. The other two expressions can be represented by:

$$S_i(y^{t+1,j}, x^{t+1,j}|C, S) = MIN_{\xi z}\xi, \quad (14)$$

subject to

$$y_{jm}^{t+1} \le \Sigma_{j=1}^J z_j y_{jm}^t, \quad m = 1, 2, \ldots, M,$$

$$\Sigma_{j=1}^J z_j x_{jn}^t \le \xi x_{jn}^{t+1}, \quad n = 1, 2, \ldots, N,$$

$$z_i \ge 0, j = 1, 2, \ldots, J,$$

and

$$S_i^{t+1}(y^{t,j}, x^{t,j}|C, S) = MIN_{\xi z}\xi, \quad (15)$$

subject to

$$y_{jm}^t \le \Sigma_{j=1}^J z_j y_{jm}^{t+1}, \quad m = 1, 2, \ldots, M,$$

$$\Sigma_{j=1}^J z_j x_{jn}^{t+1} \le \xi x_{jn}^t, \quad n = 1, 2, \ldots, N,$$

$$z_i \ge 0, \quad j = 1, 2, \ldots, J.$$

Using this school-specific productivity index, it is possible to attribute the productivity growth to either changes in efficiency across time or shifts in the frontier due to technical innovations. Further estimations of productivity under various returns to scale and disposability assumptions help find the sources and components of dynamic productivity changes, such as pure technical efficiency changes, scale efficiency changes, congestion changes, and technical innovations.

IV. Data[3]

This study uses input and output data from 36 secondary school districts in Utah for the period 1993-95. These data were collected from publications by the Utah Education Association (1992-95), the Utah Foundation (1992-95), and the Utah State Office of Education (1992-95 a, b). The variables used as school inputs in our study include: (1) student/teacher ratio based on average daily membership, (2) percentage of teachers with an MA or PhD degree, (3) expenditure per "average daily membership" (ADM) other than staff salary, (4) net assessed value per ADM, and (5) percentage of student population buying their own lunch. While variables (1) through (3) measure instructional inputs, variable (4) is a proxy for environmental input as it measures the economic condition of the neighborhood, and variable (5) is a proxy for family income of student. All these variables are aggregated over the school districts.

Our output measure is based on the standardized test administered by the state at the 11th grade in each of the school districts. This test consists of two parts—the basic battery test (a composite score of mathematics, language/English, and reading) and subject tests. Hence, the outputs are defined as the average scores obtained by each school district in (1) the basic battery test, (2) mathematics, (3) reading, (4) language/English, (5) science, and (6) social science (Utah Foundation 1995).

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[3]The data for this study are drawn from the dissertation work of Kalyan Chakraborty.

V. Analysis of Results[4]

Although the primary empirical approach utilized in this study is nonparametric, we utilize parametric estimates of the educational production function as a starting point for the DEA specification. Towards this end, we estimated a translog production function relating educational output (battery test score as a proxy for multiple-learning outputs) and the multiple inputs. Due to severe multicollinearity problems, we reestimated the model using a stepwise regression technique.[5] The parametric estimates supported the model specification chosen for the DEA analysis. We specify school units as firms producing the five learning outputs (as measured by test scores of reading, language/English, science, mathematics, and social science) from five inputs (student/teacher ratio, percentage of teachers with an MA or PhD degree, expenditure per student other than teacher's salary, net assessed value per student, and percentage of the student population buying their own lunch).

Conditional and Unconditional Technical Efficiency Estimates: Identifying the Impact of Socioeconomic and Environmental Factors

We initially estimated the basic DEA model using the five learning outputs and only the endogenous inputs in the system. The technical efficiency scores from this basic model, assuming variable returns to scale, are reported in column 3 of Table 1 (unconditional technical efficiency (T.E.) scores). Following these unconditional T.E. scores, we find 14 districts (Alpine through Granite) to be fully efficient, while school districts, such as Kane and Tintic, appear on the other end of the spectrum with efficiency levels below the mean of 0.937.

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[4]All DEA estimations were programmed in GAUSS.

[5]The stepwise regression estimates of the translog production function are available upon request.

Table 1. Conditional and Unconditional Technical Efficiency Scores

SL #	Unconditional Ordering	Unconditional T.E.	Tobit Residuals	Conditional Ordering	Conditional T.E.
1	Alpine	1.000	0.031	Alpine	1.000
2	Beaver	1.000	0.056	Tooele*	1.000
3	N. Sanpete	1.000	0.064	Rich	1.000
4	Rich	1.000	0.101	San Juan*	1.000
5	S. Summit	1.000	0.033	S. Sanpete*	1.000
6	Grand	1.000	0.087	S. Summit	1.000
7	Garfield	1.000	0.071	Tintic*	1.000
8	Uintah	1.000	0.092	Wasatch	1.000
9	Wasatch	1.000	0.020	Uintah	1.000
10	Daggett	1.000	0.043	Beaver	1.000
11	Washington	1.000	0.045	Washington	1.000
12	Logan	1.000	0.032	Salt Lake City*	1.000
13	Millard	1.000	0.065	Ogden*	1.000
14	Granite	1.000	0.048	Provo*	1.000
15	Cache	0.994	0.022	Logan	1.000
16	Davis	0.994	0.023	N. Sanpete	1.000
17	Iron	0.983	0.044	Nebo*	1.000
18	Nebo	0.979	0.019	Millard	1.000
19	Box Elder	0.974	0.015	Grand	1.000
20	Weber	0.962	-0.005	Box Elder*	1.000
21	Carbon	0.957	0.028	Cache*	1.000
22	Tooele	0.954	0.007	Daggett	1.000
23	Duchesne	0.947	0.050	Davis*	1.000
24	Jordan	0.940	-0.035	Duchesne*	1.000
25	Provo	0.936	0.009	Garfield	1.000
26	Murray	0.932	-0.038	Granite*	1.000
27	Salt Lake C.	0.925	0.034	Iron*	1.000
28	S. Sanpete	0.903	-0.005	Weber	0.968
29	San Juan	0.882	0.066	Carbon	0.959
30	Juan	0.876	-0.089	Jordan	0.940
31	Ogden	0.834	-0.017	Murray	0.932
32	Sevier	0.826	-0.106	Sevier	0.897
33	Emery	0.825	-0.125	Juab	0.884
34	Kane	0.766	-0.145	Piute	0.829
35	Piute	0.692	-0.183	Emery	0.825
36	Tintic	0.654	-0.211	Kane	0.794
	Mean T.E.	0.937	0.004		0.973

*Districts with large socioeconomic and environmental influences.

However, these efficiency scores from the controllable input DEA model do not exactly reflect inefficient management and resource wastage scenarios that would initiate correctional policy action. These scores provide little insight into the optimization behavior of school performances, which, in the real world, are highly conditioned on socioeconomic and environmental factors. Unconditional technical efficiency estimates would identify schools struggling with students from low socioeconomic backgrounds as being technically inefficient. That would lead to biased policy implications, moreso as these variables are exogenous to the school. In order to derive a robust ordering of relative conditional technical efficiencies, we estimate a DEA model incorporating both controllable and uncontrollable inputs. This model computes efficiency over the controllable factors only, but this efficiency is explicitly conditioned on school-specific socioeconomic constraints. The ordering and results from this subvector efficiency model assuming variable returns to scale is given in columns 5 and 6 of Table 1 (conditional T.E. scores). In comparing these final conditional technical efficiency scores with the unconditional scores, several important patterns emerge. The conditional ordering of schools in column 5 reveals that the Alpine through Iron school districts are fully efficient while the Weber through Kane school districts are technically inefficient. Average efficiencies over all the school districts increase by 0.036, compared to the conditional measure.

A classic example of the importance of socioeconomic factors in efficiency measures is the Tintic school unit, which ranks lowest in the unconditional measure but is among the most efficient in the conditional measure. Schools marked with an asterisk represent "inefficient" schools, which are rendered efficient once one incorporates the richness of socioeconomic and environmental factors.

In order to get additional insights on the effects of socioeconomic factors, we analyzed the residuals from a Tobit regression of the DEA efficiency scores (column 4) on socioeconomic and environmental factors. The Tobit residuals depict how a school's efficiency measure compares with an average district facing the same socioeconomic factors. The unconditional orderings are shown in column 2. School districts with larger Tobit residuals signal higher technical efficiencies given socioeconomic and environmental variables. Schools, such as Carbon, Duchesne, Provo, Salt Lake City, S. Sanpete, and San Juan, which were found to be inefficient under the unconditional measure and efficient under the conditional measure, have relatively larger residuals associated with them, signaling that socioeconomic factors play a major role in their deviation from the frontier.

Scale Errors: The Scale Component in Conditional Technical Efficiency Estimates

We estimated the conditional DEA model under varying returns-to-scale structures. Table 2 (columns 3 through 8) reports the conditional efficiency scores and rankings of school districts as observed under constant, nonincreasing, variable returns to scale (CS, NS, and VS, respectively), and free disposability (of inputs) assumptions. A brief description of these results follows.

A comparison of efficiency scores between CS and NS reveals no significant changes except for the Daggett school district, which turned into one of the most efficient districts under NS. As expected, almost all of the school districts improved their efficiency scores under the VS assumption, however, substantial improvements are evident in the cases of Piute and Sevier, where efficiency scores increased by 35 and 31 percent, respectively. Under the VS assumption, the least efficient Kane school district is about 24 percent inefficient, implying that a student would have a 24 percent higher chance of making better achievement scores if the school district would have used its resources more efficiently, given the socioeconomic status of its students and the environmental factors within which it operates. Overall, efficiency rankings change as the returns to scale assumptions were varied.

Table 2. Conditional Technical Efficiency Scores with Varying Returns to Scale and Input Possibilities

S.L.#	School Districts	CRS T.E.	CRS Rank	NRS T.E.	NRS Rank	VRS T.E.	VRS Rank	Scale Index μ	Int. Index δ	Weak Disp. VRS. T.E.	Cong. Index β
1	Alpine	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
2	Beaver	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
3	Box Elder**	0.925	8	0.925	8	1.000	1	0.925	1.000	1.000	1.000
4	Cache	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
5	Carbon*	0.899	11	0.902	10	0.959	3	0.938	0.997	0.959	1.000
6	Daggett*	0.690	16	1.000	1	1.000	1	0.690	0.690	1.000	1.000
7	Davis	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
8	Duchesne	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
9	Emery*	0.716	15	0.721	14	0.825	9	0.867	0.993	0.825	1.000
10	Garfield	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
11	Grand	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
12	Granite*	0.940	6	0.940	5	1.000	1	0.940	1.000	1.000	1.000
13	Iron	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
14	Jordan*‡	0.926	7	0.939	6	0.940	4	0.985	0.986	0.972	0.967
15	Juab*‡	0.817	13	0.818	12	0.884	7	0.924	0.999	0.892	0.991
16	Kane*	0.765	14	0.766	13	0.794	10	0.963	0.999	0.794	1.000
17	Millard	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
18	Nebo**	0.996	2	0.996	2	1.000	1	0.996	1.000	1.000	1.000
19	N. Sanpete**	0.977	3	0.977	3	1.000	1	0.977	1.000	1.000	1.000
20	Piute**	0.615	18	0.615	16	0.829	8	0.742	1.000	0.829	1.000
21	Rich	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
22	San Juan	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
23	Sevier**‡	0.685	17	0.685	15	0.897	6	0.763	1.000	0.900	0.997
24	S. Sanpete	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
25	S. Summit*	0.964	5	1.000	1	1.000	1	0.964	0.964	1.000	1.000
26	Tintic	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
27	Tooele*	0.966	4	0.966	4	1.000	1	0.966	1.000	1.000	1.000
28	Uintah	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
29	Wasatch*	0.852	12	0.874	11	1.000	1	0.852	0.974	1.000	1.000
30	Washington	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
31	Weber**‡	0.917	9	0.917	9	0.968	2	0.947	1.000	1.000	0.968
32	Salt Lake C.	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
33	Ogden	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
34	Provo	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
35	Logan	1.000	1	1.000	1	1.000	1	1.000	1.000	1.000	1.000
36	Murray*‡	0.903	10	0.926	7	0.932	5	0.969	0.976	0.954	0.977
	Mean T.E.	0.932		0.944		0.973					

* → ($\mu < 1, \delta < 1$) ** → ($\mu < 1, \delta = 1$) ‡ → ($\beta < 1$).

In order to identify scale components from the conditional technical efficiency measure, we construct a scale efficiency index, μ, which is the ratio of the values of the objective functions evaluated at the optimum, from CS and VS efficiency measures. The results are reported in column 9 of Table 2. For example, the Alpine and Logan school districts, with a value of $\mu = 1$, are scale efficient as they are equally efficient regardless of the scale assumptions. For schools with a value of $\mu < 1$, e.g., Box Elder and Daggett, imply that they are not scale efficient. Once the scale-inefficient units have been identified, in order to enforce any kind of corrective actions we need to know if the input scale inefficiency is due to production of an inefficiently small output in the realm of increasing returns to scale or due to the production of an inefficiently large output in the realm of a decreasing returns to scale. For this purpose, we construct an intermediate index, δ, the ratio of the efficiency scores from optimally evaluated objective functions under the constant and nonincreasing returns-to-scale assumptions. Thus, for any district for which the relationship ($\mu < 1$ and $\delta = 1$) holds, input scale inefficiency is due to increasing returns to scale. Such schools are marked with an * in Table 2. Thus, the Box Elder school district is scale inefficient due to increasing returns to scale. The policy implications would be to restrict output-increasing input changes in that school. If for any school $\mu < 1$ and $\delta < 1$, then the input scale inefficiency is due to decreasing returns to scale. For example, Provo, Carbon, Daggett, and Emery need to decrease output, as they are producing an inefficiently large output vector in a region of decreasing returns to scale. Schools that are scale inefficient in a decreasing returns to scale are marked with ** in Table 2.

Congesting Inputs: Does Teacher Overeducation Really Decrease Student Education?

There is ample evidence in the literature (Tsang and Levin 1985) that shows overutilization of teachers with advanced degrees might actually reduce student achievement scores. According to these theoretical tunes, we test the hypotheses that increases in the percentage of teachers with a PhD or MA might lead to input congestion instead of a monotonic increase in output for given scales. Towards this end, we specify the input, teachers with advanced degrees, as weakly disposable. The conditional technical efficiency scores obtained under the assumptions of weak disposability of the teaching input, free disposability of all other inputs, and variable returns to scale are reported in columns 11 and 12 of Table 2. It is evident from column 11 that there have been marginal improvements in the efficiency scores of a few school districts (when compared with the free disposable VRS measure in column 7), however, substantial improvements are noticeable in the cases of Jordan and Weber whose efficiency increased from 0.940 and 0.968 to 0.972 and 1, respectively. This suggests that the presence of input congestion, which is responsible for lower efficiency scores. On the other hand, Kane's poor efficiency scores, which do not change under weak disposability assumptions, cannot be blamed on overqualified teachers.

In order to assess school-specific congestions, we construct an input congestion index β, which yields a comparison of feasible input shrinkage under weak and freely disposable inputs. It is formed from the optimally evaluated VF and VW conditional measures of technical efficiency. Thus, if the congestion index is $\beta = 1$, then the input subvector (i.e., teachers with advanced degrees) does not congest the output vector of student's achievement scores in that school. But if $\beta < 1$, then the input subvector congests output. In our study, the Jordan, Juab, Sevier, Weber, and Murray school districts are overcrowded with MAs and PhDs which congest student's productivity. Schools for which teachers' overeducation decreases students' learning are marked with a ‡ in Table 2.

Input Slacks: Nonradial Conditional Technical Efficiency Estimates

In order to eliminate technical efficiency estimate biases due to the presence of input slacks, we estimate a nonradial conditional technical efficiency measure. The efficiency scores obtained from the conditional radial and nonradial measures (CF) are reported in Table 3. Most school districts become less efficient under the nonradial measure, and the mean efficiency goes down by 0.038. This is not surprising, since nonradial measures are constructed under a more restrictive definition of technical efficiency. Schools that were not radially projected back onto the efficient subset of the isoquant due to the presence of input slacks are marked with an * in Table 3, and their true conditional technical efficiencies after eliminating input slacks are reported in column (5).

Dynamic Models: The Need for Research or the Need for Extension in Education

Using multiperiod analysis, intertemporal input correspondences are defined to derive estimates for changes in productivity over the time period 1993-95. Column 3 of Table 4 reports the Malmquist productivity indexes for this time period. Schools that have a productivity index less than unity have had improvements in productivity over time.[6] Thus, school districts such as Box Elder, Davis, and Duchesne (marked with an * in Table 4) have experienced aggregate productivity growths across the time period 1993-95. Values greater than unity for schools such as Alpine and Beaver have experienced decreases in productivity.

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[6]Note that these input based scores do not have an upper bound of unity. Remember from equations (14) and (15) that as the conditions x^{t+1} $L^t(y^{t+1})$ and x^t $L^{t+1}(y^t)$ do not necessarily hold, the solutions to the multiperiod efficiency parameters may have upper bounds greater than unity.

Table 3. Radial and Nonradial, Conditional Technical Efficiency Scores

S.L.#	School Districts	Radial T.E.	Radial Rank	Nonradial T.E.	Nonradial Rank
1	Alpine	1.000	1	1.000	1
2	Beaver	1.000	1	1.000	1
3	Box Elder*	0.925	8	0.869	6
4	Cache	1.000	1	1.000	1
5	Carbon*	0.899	11	0.849	7
6	Daggett*	0.690	16	0.590	17
7	Davis	1.000	1	1.000	1
8	Duchesne*	1.000	1	1.000	1
9	Emery	0.716	15	0.678	15
10	Garfield	1.000	1	1.000	1
11	Grand	1.000	1	1.000	1
12	Granite*	0.940	6	0.930	5
13	Iron	1.000	1	1.000	1
14	Jordan*	0.926	7	0.833	8
15	Juab*	0.817	13	0.755	12
16	Kane*	0.765	14	0.715	14
17	Millard	1.000	1	1.000	1
18	Nebo*	0.996	2	0.952	2
19	N. Sanpete*	0.977	3	0.833	9
20	Piute*	0.615	18	0.505	18
21	Rich	1.000	1	1.000	1
22	San Juan	1.000	1	1.000	1
23	Sevier*	0.685	17	0.647	16
24	S. Sanpete	1.000	1	1.000	1
25	S. Summit*	0.964	5	0.938	4
26	Tintic	1.000	1	1.000	1
27	Tooele*	0.966	4	0.946	3
28	Uintah	1.000	1	1.000	1
29	Wasatch*	0.852	12	0.751	13
30	Washington	1.000	1	1.000	1
31	Weber*	0.917	9	0.794	11
32	Salt Lake C.	1.000	1	1.000	1
33	Ogden	1.000	1	1.000	1
34	Provo	1.000	1	1.000	1
35	Logan	1.000	1	1.000	1
36	Murray*	0.903	10	0.822	10
	Mean T.E.	0.932		0.900	

*Districts with input slacks in the radial measure.

Table 4. Malmqusit Productivity Index 1993-95: Pure Technical Efficiency (τ), Scale Efficiency (ψ), Congestion (ϵ), and Frontier Shifts (Ω)

S.L.#	School District	Malmquist Prod. Index	Efficiency Change	Technical Change	τ	ψ	ϵ	Ω
1	Alpine	1.146	1.000	1.146	1.000	1.000	1.000	1.146
2	Beaver	1.111	0.865	1.285	0.865	1.000	1.000	1.285
3	Box Elder*	0.962	0.946	1.016	1.017	0.928	1.002	1.016
4	Cache	1.374	0.943	1.457	1.000	0.937	1.006	1.457
5	Carbon	1.029	0.803	1.282	0.803	0.999	1.000	1.282
6	Davis*	0.985	0.900	1.094	0.868	1.033	1.004	1.094
7	Duchesne*	0.883	0.671	1.316	0.761	0.882	1.000	1.316
8	Emery	1.031	0.839	1.229	0.734	1.144	1.000	1.229
9	Garfield*	0.721	0.566	1.275	0.608	0.930	1.000	1.275
10	Grand	1.571	1.000	1.571	1.000	1.000	1.000	1.571
11	Granite*	1.000	0.922	1.084	0.873	1.056	1.000	1.084
12	Iron	1.223	0.911	1.342	0.951	0.958	1.000	1.342
13	Jordan	1.549	1.034	1.498	1.028	0.972	1.035	1.498
14	Juab*	0.870	0.802	1.085	0.785	1.011	1.009	1.085
15	Kane	1.304	1.098	1.187	1.306	0.841	1.000	1.187
16	Millard*	0.857	0.694	1.234	0.787	0.882	1.000	1.234
17	Nebo	1.050	0.931	1.128	0.918	1.012	1.002	1.128
18	N. Sanpete	1.007	0.770	1.307	0.821	0.938	1.000	1.307
19	Piute	2.598	1.918	1.354	1.444	1.328	1.000	1.354
20	Rich*	0.985	0.773	1.273	1.000	0.773	1.000	1.273
21	San Juan*	0.737	0.482	1.529	0.616	0.782	1.000	1.529
22	Sevier	1.045	0.909	1.149	0.771	1.174	1.004	1.149
23	S. Sanpete	1.214	0.992	1.224	1.001	0.991	1.000	1.224
24	S. Summit*	0.738	0.536	1.378	0.546	0.981	1.000	1.378
25	Tooele	1.019	0.761	1.340	0.778	0.977	1.000	1.340
26	Uintah	1.354	1.000	1.354	1.000	1.000	1.000	1.354
27	Wasatch	1.216	1.080	1.126	1.000	1.080	1.000	1.126
28	Washington	1.371	1.000	1.371	1.000	1.000	1.000	1.371
29	Weber*	0.912	0.784	1.163	0.775	0.974	1.039	1.163
30	Salt Lake C.	1.000	0.819	1.221	0.821	0.926	1.078	1.221
31	Ogden*	0.973	0.858	1.134	0.783	1.096	1.000	1.134
32	Provo	1.121	0.835	1.342	1.068	0.782	1.000	1.342
33	Logan	1.004	0.974	1.031	1.000	0.974	1.000	1.031
34	Murray*	0.983	0.860	1.143	0.849	0.990	1.024	1.143
	Mean T.E.	1.116	0.890	1.255	0.899	0.981	1.006	1.255

*School districts with net productivity increases during the time period 1993-1995.

However, this reflects aggregate changes in the schools' productivity over the given period of time. This movement of the school may be visualized as a series of positive effects which move the school unit through time. Similarly, schools with decreases in productivity, as signaled by index values greater than unity, experience dominating negative effects. Initially, we identify two components of the dynamic change in school productivity. The first component is due to changes in technical efficiency. The second component of productivity is due to an actual shift of the frontier across time. These components of the grand productivity change, if less than unity, are sources of the dynamic productivity improvement. Analysis of the contribution of efficiency to dynamic productivity increases for school districts (column 5) yields interesting conclusions. Most schools have become more technically efficient over time and draw their increases in productivity from these efficiency improvements. More interesting is the pattern displayed in an analysis of frontier shifts across the time period. Technological innovations have no contribution to the productivity increase across the time period 1993-95, and increases in productivity are attributed solely to efficiency increases. Thus, this study demonstrates an urgent need for research and technological innovation in education.

These conclusions are examined in greater detail by a more comprehensive decomposition of the productivity index. Towards this end, we identify all four components which derive dynamic productivity differences and evaluate the contribution of each to the change in productivity. These four components are changes in pure technical efficiency (τ), changes in scale efficiency (ψ), changes in congestion (ϵ), and changes in the frontier itself, i.e., technical innovations (Ω). Table 4 reports these four sources of productivity change. We find changes in scale and changes in technical efficiency to be the primary contributors to the school districts' productivity changes over time. Interestingly, the robustness of earlier conclusions regarding technical innovations is further established. Both congestion and technical change have no positive contribution on productivity increases over the period 1993-95. This result clearly provides insights into incorrect policy actions generated by a failure to distinguish between technical and productivity changes. While these two concepts are often treated synonymously in empirical literature, analytically, they are separate and could move in opposite directions as demonstrated in this analysis.

VI. Sensitivity Analysis

The nonparametric approach of DEA analysis offers strong advantages over parametric ones, especially by imposing only minimal structure on the specified technology. However, with the inability to conduct tests of significance, the robustness of the frontier estimates and deviations from it needs to be established. We establish robustness of efficiency estimates by performing two kinds of sensitivity analysis. The conditional DEA model was reestimated with (1) the exclusion of the expenditure input (x_3), and (2) the exclusion of the math score output (y_1). A comparison of the original DEA model with these reestimates provides a test of the robustness of the results derived in this study. These results are reported in Table 5. Based on these comparisons, our results from earlier models remain unchanged. In both cases, the correlation between the original and new measures were above 0.965. Thus, conclusions drawn from this analysis are quite reliable.

VII. Discussion and Conclusions

In this paper, we estimated the deviations of 36 Utah school districts from a technically efficient frontier during the year 1995. Identification of the components of these deviations as scale, congestion, and pure technical efficiency provide additional insights into the sources of deviation

Table 5. Sensitivity Analysis

SL. #	School District	Conditional T.E.	Conditional T.E. w/o Input (x_3)	Conditional T.E. w/o Output (y_1)
1	Alpine	1.000	1.000	1.000
2	Beaver	1.000	1.000	0.937
3	Box Elder	0.925	0.925	0.925
4	Cache	1.000	1.000	1.000
5	Carbon	0.899	0.899	0.860
6	Daggett	0.690	0.690	0.690
7	Davis	1.000	0.937	1.000
8	Duchesne	1.000	1.000	1.000
9	Emery	0.716	0.716	0.677
10	Garfield	1.000	1.000	1.000
11	Grand	1.000	1.000	1.000
12	Granite	0.940	0.913	0.940
13	Iron	1.000	0.967	0.956
14	Jordan	0.926	0.909	0.900
15	Juab	0.817	0.817	0.817
16	Kane	0.765	0.670	0.686
17	Millard	1.000	1.000	1.000
18	Nebo	0.996	0.993	0.996
19	N. Sanpete	0.977	0.977	0.977
20	Piute	0.615	0.615	0.549
21	Rich	1.000	1.000	1.000
22	San Juan	1.000	1.000	1.000
23	Sevier	0.685	0.685	0.685
24	S. Sanpete	1.000	1.000	1.000
25	S. Summit	0.964	0.964	0.931
26	Tintic	1.000	1.000	1.000
27	Tooele	0.966	0.954	0.966
28	Uintah	1.000	1.000	1.000
29	Wasatch	0.852	0.852	0.838
30	Washington	1.000	0.998	1.000
31	Weber	0.917	0.909	0.917
32	Salt Lake City	1.000	1.000	1.000
33	Ogden	1.000	1.000	1.000
34	Provo	1.000	1.000	1.000
35	Logan	1.000	1.000	1.000
36	Murray	0.903	0.869	0.758
	Mean T.E.	0.932	0.924	0.917

and lays out the path for exact policy formulation. All efficiency estimations (both radial and nonradial) carefully account for confounding socioeconomic and environmental factors, which have been the source of intense debate in the education efficiency literature. Further, we addressed the controversial issue of teacher overqualification as an impediment to student learning. From a dynamic perspective, we estimated the changes in productivity of Utah school districts as observed over the time period 1993-95. For the dynamic analysis, we decomposed the aggregate productivity change measure into scale, congestion, pure technical efficiency, and technical innovation changes in order to locate the sources of the productivity change over time. This study is a first attempt at assessing the dynamic productivity of Utah school districts. The results of this study provides several important insights into educational efficiency and a basis for correctional policy action.

We find strong evidence of technical efficiency for most of the schools in the sample with mean efficiency scores above 90 percent. Even with the stricter nonradial measures, mean efficiency levels for Utah schools remain at the 90 percent level. Thus, the data support the conclusion that schools are technically efficient, and additional productivity can be secured only through technical innovations (perhaps involving increases in overall expenditures). The estimates provide a strong evidence of large socioeconomic and environmental influences on technical efficiency scores. Scale inefficiencies are observed for a fairly large number of schools, which suggests policy action with regard to input and output uses for those schools. The Jordan, Juab, Sevier, Weber, and Murray school districts are found to be overutilizing their highly qualified teacher input, leading to a congestion of student learning. The dynamic analysis results support the earlier conclusions. Schools that have had net increases in productivity over the time period 1993-95 have done so mostly due to increases in efficiency. Surprisingly, technical progress in Utah schools has been slow, with many schools experiencing inwards shifts in their technical frontiers over the time period.

Both static and dynamic estimates strongly suggest policies geared toward educational research rather than extension. In order to secure increases in educational productivity in Utah schools, which are technically efficient, policies should focus on research expenditures for the introduction of technological innovations in public education.

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